

Curriculum Vitae

Si-Qi Liu

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I. Personal Information

- Name: Si-Qi Liu
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- Nationality: Chinese
- Sex: Male

II. Education

- 1999-2003, Department of Physics, Tsinghua University, BS
- 2003-2007, Department of Mathematical Sciences, Tsinghua University, PhD
Dissertation: Bihamiltonian Structures of PDEs
Supervisor: Youjin Zhang

III. Working Experience

- 2007-2009, Department of Mathematical Sciences, Tsinghua University, Assistant Researcher
- 2009-2010, Department of Mathematical Sciences, Tsinghua University, Lecturer
- 2010-2015, Department of Mathematical Sciences, Tsinghua University, Associate Professor
- 2015-present, Department of Mathematical Sciences, Tsinghua University, Professor

IV. Research

My research is focused on the classification of integrable evolutionary PDEs under the action of various coordinate transformations, and the applications of the corresponding classification theorems to other areas in mathematical physics, such as Gromov-Witten invariants theory, singularity theory, and so on.

In 2001, Dubrovin and Zhang developed an axiomatic approach to the relation between semisimple Gromov-Witten invariant theory and integrable hierarchy theory. They proposed to study the classification problem of semisimple bihamiltonian structures under the action of Miura type transformations.

In 2005-2006, we showed that the equivalence class of a semisimple bihamiltonian structure modulo Miura type transformations is uniquely determined by its leading term and a collection of one-variable functions. We named these functions the central invariants of the bihamiltonian structure. Our result gives the uniqueness part of the classification problem. Then it is natural to ask the existence part: given a bihamiltonian structure of hydrodynamic type and a collection of one-variable functions, does there exist a deformation of this bihamiltonian structure with those functions as its central invariants?

The uniqueness part is solved by computing the corresponding bihamiltonian cohomologies. The existence part also corresponds certain bihamiltonian cohomologies, which are very difficult to compute. We tried to solve the existence problem by using reciprocal transformations, which are a kind of coordinate transformations that not only transform dependent variables but also transform independent variables. Reciprocal transformations may destroy the locality of Hamiltonian structures. To resolve this problem, one must develop a general theory of nonlocal Hamiltonian structure. In 2011, we managed to give such a theory for a special kind of reciprocal transformations. The resulting geometric structure is a generalization of the Jacobi structure in the finite-dimensional Poisson geometry, so we named it the Jacobi structure for partial differential equations. We also proved a Darboux theorem for these Jacobi structures.

The theory of Jacobi structures is not enough to prove the existence theorem, but, when developing the theory, we learned many interesting facts about the infinite jet space of the odd cotangent bundle of a manifold, which are very useful for the computation of bihamiltonian cohomologies. In 2013, we developed a new framework for the existence problem, and proved that the existence theorem is true for the bihamiltonian structure of the KdV hierarchy. In 2015, Carlet, Posthuma, and Shadrin proved the existence theorem for general semisimple bihamiltonian structures by using our framework and some other computation techniques. So the classification problem proposed by Dubrovin-Zhang is entirely resolved now.

In 2007, Fan, Jarvis, and Ruan introduced the FJRW theory, and proved the ADE Witten conjecture, which states that the partition function of FJRW theory for an ADE singularity is a tau function of the corresponding Drinfeld-Sokolov hierarchy. It is natural to expect that the similar statement should be true for BCF boundary singularities and the corresponding Drinfeld-Sokolov hierarchy. In 2008, we computed the central invariants of the bihamiltonian structures for Drinfeld-Sokolov hierarchies. These central invariants are proportional to the square of lengths of the simple coroots of the corresponding simple Lie algebras, which implies that the Drinfeld-Sokolov hierarchies of BCFG type can never be equivalent to a cohomological field theory, so our result gave a negative answer to the above problem. In 2015, we give the correct construction of the FJRW theory for BCFG type, which is a partial cohomological field theory, and proved the corresponding Witten conjecture.

Recently, motivated by another more general classification program, we began to study Hodge integrals and the corresponding integrable hi-

erarchies. In 2015, we construct such an integrable hierarchy for Hodge integrals of each semisimple cohomological field theory, and find many interesting conjectures for the simplest case with the help of computer programs. In particular, we find a conjectural relation between a kind of special cubic Hodge integral and the Volterra hierarchy. Then Dubrovin and Yang extend this conjecture to include the Gaussian unitary ensemble model as the third part. We proved these conjectures in 2016, and then find its generalization for all special cubic Hodge integrals. The integrable hierarchy appearing in this generalization seems new, so we name it the *Fractional Volterra Hierarchy*, since it can be regarded as a fractional generalization of the Volterra hierarchy.

V. Publications and preprints

1. Liu, Si-Qi; Yang, Di; Zhang, Youjin; Zhou, Chunhui. The loop equation for special cubic Hodge integrals. Preprint arXiv: 1811.10234.
2. Liu, Si-Qi; Wu, Chao-Zhong; Zhang, Youjin Zhang; Zhou, Xu. Drinfeld-Sokolov hierarchies and diagram automorphisms of affine Kac-Moody algebras. Preprint arXiv: 1811.10137.
3. Liu, Si-Qi; Zhang, Youjin; Zhou, Chunhui. Fractional Volterra hierarchy. *Lett. Math. Phys.* 108 (2018), 261–283.
4. Dubrovin, Boris; Liu, Si-Qi; Zhang, Youjin. Bihamiltonian cohomologies and integrable hierarchies II: the tau structures. *Comm. Math. Phy.* 361 (2018), 467–524.
5. Dubrovin, Boris; Liu, Si-Qi; Yang, Di; Zhang, Youjin. Hodge-GUE correspondence and the discrete KdV equation. Preprint arXiv: 1612.02333.
6. Liu, Si-Qi; Zhang, Youjin; Zhou, Xu. Central invariants of the constrained KP hierarchies. *J. Geom. Phys.* 97 (2015), 177–189.
7. Dubrovin, Boris; Liu, Si-Qi; Yang, Di; Zhang, Youjin. Hodge integrals and tau-symmetric integrable hierarchies of Hamiltonian evolutionary PDEs. *Adv. Math.* 293 (2016), 382–435.
8. Liu, Si-Qi; Ruan, Yongbin; Zhang, Youjin. BCFG Drinfeld-Sokolov hierarchies and FJRW-theory. *Invent. Math.* 201 (2015), 711–772.
9. Liu, Si-Qi; Yang, Di; Zhang, Youjin. Uniqueness theorem of \mathcal{W} -constraints for simple singularities. *Lett. Math. Phys.* 103 (2013), 1329–1345.
10. Fu, Yulong; Liu, Si-Qi; Zhang, Youjin; Zhou, Chunhui. Proof of a conjecture on the genus two free energy associated to the A_n singularity. *J. Geom. Phys.* 76 (2014), 10–24.
11. Liu, Si-Qi; Zhang, Youjin. Bihamiltonian cohomologies and integrable hierarchies I: a special case. *Comm. Math. Phys.* 324 (2013), 897–935.
12. Dubrovin, Boris; Liu, Si-Qi; Zhang, Youjin. On the genus two free energies for semisimple Frobenius manifolds. *Russ. J. Math. Phys.* 19 (2012), 273–298.

13. Liu, Si-Qi; Xu, Dingdian; Zhang, Youjin. The inversion symmetry of the WDVV equations and tau functions. *Phys. D* 241 (2012), 2168–2177.
14. Liu, Si-Qi; Wu, Chao-Zhong; Zhang, Youjin. On the Drinfeld-Sokolov hierarchies of D type. *Int. Math. Res. Not. IMRN* 2011, 1952–1996.
15. Liu, Si-Qi; Zhang, Youjin. Jacobi structures of evolutionary partial differential equations. *Adv. Math.* 227 (2011), 73–130.
16. Chen, Ming; Liu, Si-Qi; Zhang, Youjin. Hamiltonian structures and their reciprocal transformations for the r -KdV-CH hierarchy. *J. Geom. Phys.* 59 (2009), 1227–1243.
17. Liu, Si-Qi; Wu, Chao-Zhong; Zhang, Youjin. On properties of Hamiltonian structures for a class of evolutionary PDEs. *Lett. Math. Phys.* 84 (2008), 47–63.
18. Dubrovin, Boris; Liu, Si-Qi; Zhang, Youjin. Frobenius manifolds and central invariants for the Drinfeld-Sokolov bihamiltonian structures. *Adv. Math.* 219 (2008), 780–837.
19. Liu, Si-Qi; Zhang, Youjin. On quasi-triviality and integrability of a class of scalar evolutionary PDEs. *J. Geom. Phys.* 57 (2006), 101–119.
20. Chen, Ming; Liu, Si-Qi; Zhang, Youjin. A two-component generalization of the Camassa-Holm equation and its solutions. *Lett. Math. Phys.* 75 (2006), 1–15.
21. Dubrovin, Boris; Liu, Si-Qi; Zhang, Youjin. On Hamiltonian perturbations of hyperbolic systems of conservation laws. I. Quasi-triviality of bi-Hamiltonian perturbations. *Comm. Pure Appl. Math.* 59 (2006), 559–615.
22. Liu, Si-Qi; Zhang, Youjin. On the quasitriviality of deformations of bihamiltonian structures of hydrodynamic Type. Preprint arXiv: math/0406626.
23. Liu, Si-Qi; Zhang, Youjin. Deformations of semisimple bihamiltonian structures of hydrodynamic type. *J. Geom. Phys.* 54 (2005), 427–453.